AIAA Journal

VOLUME 8 APRIL 1970 NUMBER 4

Transient and Steady Heat Transfer in a Conducting and Radiating Medium

YAN-PO CHANG* AND CHE-SHING KANG†
State University of New York at Buffalo, Buffalo, N.Y.

The transient and steady heat transfer in a conducting, emitting, and absorbing medium is analyzed according to the differential formulation. The differential equations are first transformed into integral equations and then solved by successive approximations. Great insight into the effect of absorption and emission on the temperature distribution is afforded by the calculation of radiation potential. The steady problem is briefly examined and discussed together with the transient problem. For small times emission is found negligibly small in comparison with absorption, whereas for large times it is important only in a thin layer near the hotter surface.

Nomenclature

c = specific heat
G = Green's function
h_s = $3\epsilon_s/[2(2-\epsilon_s)]$
I^* = intensity of radiation
k = thermal conductivity
L = thickness of medium
l = κL, optical thickness of medium
N = $k\kappa/(4\sigma T_*^3)$
n = inward drawn normal to surface
$\mathbf{q}^*$ = heat flux
$\mathbf{q}$ = $\mathbf{q}^*/(\sigma T_*^4)$
β = $3^{1/2}$
T^* = temperature
T_* = reference temperature
T = T^*/T_*
t^* = time
t = $4\kappa\sigma T_*^3 t^*/(\rho c)$
x^* = Cartesian coordinate
x = κx^*
ϵ = emissivity
σ = Stefan-Boltzmann's constant
ϕ = $\pi\bar{I}^*/(\sigma T_*^4)$
ρ = density of medium
κ = absorption coefficient
χ = $-3NT - \phi$

Subscripts

b = black
c = conduction
r = radiation
s = surface
$0,l$ = surfaces at $x = 0,l$

Superscripts

$*$ = dimensional quantity
$-$ = average, or Laplace transform

Introduction

THE problem of heat transfer in a conducting, absorbing, and emitting medium has been of considerable interest in recent years. Viskanta and Grosh[1] studied the steady-state heat transfer in a plane gray medium bounded by black surfaces according to the rigorously formulated integro-differential equation. Lick[2] investigated the steady problem through the use of kernel approximation and considered three particular cases with emphasis on the approximate calculation of rate of heat transfer. Subsequent to Lick's work a number of related papers have appeared in the literature; for instance, Greif[3] and Wang and Tien.[4] Einstein[5] attacked the problem by using zonal exchange factors. Howell[6] employed the exchange factor approximation to investigate this type of problem.

For the transient state, there have been only a few investigations. Lick[7] considered the linearized problem of transient energy transfer in a semi-infinite medium bounded by a non-emitting and nonreflecting surface. Nemchinov[8] also studied the linearized problem but was interested only in the propagation of thermal waves. Viskanta and Lall[9] attempted at first to solve the problem of transient heat transfer in a spherical medium from the integro-differential equation, but later diverted to approximate calculations.

This paper concerns mainly the transient heat transfer by combined conduction and radiation in a plane layer. The formulation of fundamental equations is based on the differential approximation reported by Traugott,[10] Cohen,[11] Cheng,[12] and Chang.[13] The boundary conditions of the radiative transfer are formulated in a manner much simpler than those reported in Refs. 12 and 13, as shown in the Appendix of this paper. The method of solution is based on

Received February 3, 1969; revision received October 22, 1969. This work was sponsored in part by the National Science Foundation, under contract GK-1726. The computer time was partly provided by the Computer Center of the University which was supported by NSF Grant 7318 and NIH Grant 00126.
* Professor of Engineering.
† Research Assistant, Department of Mechanical Engineering.

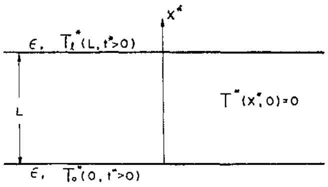

Fig. 1 Geometry of medium.

that suggested by Chang.[14] The steady problem is briefly examined and only some calculated results of radiation potential and temperature are presented to facilitate the discussion on those in transient state and to verify the accuracy of the present analysis.

Statement of the Problem

The following simplifying assumptions are made in the formulation for the energy transfer by radiation: 1) radiation is locally in thermodynamic equilibrium and scattering is negligible, 2) the medium is gray and has a refractive index of unity, 3) the surfaces are gray, and emit and reflect radiation diffusely, and 4) the radiation field is quasi-steady at any instant of time. Under these assumptions, the basic equations are presented in the Appendix.

The problem to be considered is illustrated in Fig. 1: a conducting, absorbing, and emitting medium is bounded by two infinite parallel plane surfaces. The medium is initially at absolute zero temperature. The surfaces are suddenly brought to and kept at constant and uniform temperatures. For simplicity, we assume that the physical and thermal properties of the medium are constant and that the radiation properties of the two surfaces are the same.

Formal Solutions

For transient state, the governing equations of the radiation intensity are given by Eqs. (A2) and (A5) in the Appendix. Expressing in terms of the dimensionless quantities as defined in the Nomenclature and specializing to the present problem, we obtain

$$\partial^2\phi/\partial x^2 - 3\phi = -3T^4(x,t) \tag{1}$$

$$\begin{aligned} \partial\phi/\partial x &= h(\phi - T_0^4) \text{ at } x = 0 \\ \partial\phi/\partial x &= -h(\phi - T_l^4) \text{ at } x = l \end{aligned} \tag{2}$$

The dimensionless energy equation is obtained from (A7) and (A2) by specializing to the present problem,

$$\partial T/\partial t - N\partial^2 T/\partial x^2 = \phi - T^4 \tag{3}$$

with boundary and initial conditions

$$T(0,t) = T_0,\ T(l,t) = T_l,\ T(x,0) = 0 \tag{4}$$

We wish to solve (1) for $\phi(x,t)$ and (3) for $T(x,t)$ by the method of successive approximations. For this purpose it is convenient to transform them into integral equations by considering the right-hand sides of (1) and (3) as known functions of x and t such that the method of Green's functions can be applied. If $G_\phi(x|x')$ is the Green's function associated with ϕ, the formal solution for ϕ satisfying (1) and (2) is then given by

$$\phi(x,t) = \phi_1(x) + 3\int_0^l T^4(x',t)G_\phi(x|x')dx' \tag{5}$$

In (5) the function $\phi_1(x)$ is the homogeneous solution of (1) satisfying (2),

$$\phi_1(x) = A\sinh\beta x + B\cosh\beta x \tag{6}$$

$$A = (h/\beta)(B - T_0^4)$$

$$B = \frac{hT_0^4(\beta\cosh\beta l + h\sinh\beta l) + h\beta T_l^4}{(\beta^2 + h^2)\sinh\beta l + 2h\beta\cosh\beta l}$$

where β is $3^{1/2}$ for brevity. The Green's function $G_\phi(x|x')$ can be found by writing

$$G_\phi(x|x') = (1/2\beta)e^{-\beta|x-x'|} + C_1(x')e^{-\beta x} + C_2(x')e^{\beta x}$$

Evaluating $C_1(x')$ and $C_2(x')$ by the boundary conditions

$$dG_\phi/dx = hG_\phi \text{ at } x = 0 \text{ and } dG_\phi/dx = -hG_\phi \text{ at } x = l$$

we obtain

$$G_\phi(x|x') = \frac{(\beta\cosh\beta x' + h\sinh\beta x')[\beta\cosh\beta(l-x) + h\sinh\beta(l-x)]}{\beta[(\beta^2+h^2)\sinh\beta l + 2h\beta\cosh\beta l]} \tag{7}$$

for $x > x'$. For $x < x'$ we simply interchange x and x' in (7).

The formal solution of (3) for $T(x,t)$ can be obtained in the same way. If $G_T(x,t|x',t')$ is the Green's function associated with $T(x,t)$, we obtain

$$T(x,t) = T_c(x,t) + \int_0^t\int_0^l [\phi(x',t') - T^4(x',t')]G_T(x,t|x',t')dx'dt' \tag{8}$$

In (8), $T_c(x,t)$ is the homogeneous solution of (3) satisfying (4), i.e., the solution for pure conduction,[15]

$$T_c(x,t) = T_0 - (T_0 - T_l)\frac{x}{l} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{T_l\cos n\pi - T_0}{n}\sin\frac{n\pi x}{l}\exp\left(-\frac{Nn^2\pi^2 t}{l}\right) \tag{9}$$

which is suitable for large values of Nt, or

$$T_c(x,t) = T_0\sum_{n=0}^{\infty}\left[\operatorname{erfc}\frac{2nl + x}{2(Nt)^{1/2}} - \operatorname{erfc}\frac{2(n+1)l - x}{2(Nt)^{1/2}}\right] + T_l\sum_{n=0}^{\infty}\left[\operatorname{erfc}\frac{(2n+1)l - x}{2(Nt)^{1/2}} - \operatorname{erfc}\frac{(2n+1)l + x}{2(Nt)^{1/2}}\right] \tag{9a}$$

which is suitable for small values of Nt. The Green's function, $G_T(x,t|x',t')$, can be found by eigenfunction expansion[16] or Laplace transformation.[15] The Laplace transform of $G_T(x,t|x',t')$ can be readily written down from (7) by setting $h \to \infty$ and $\beta = p^{1/2}$ as

$$\bar{G}_T(x,p|x') = \frac{\sinh p^{1/2}x'\sinh p^{1/2}(l-x)}{p^{1/2}\sinh p^{1/2}l} \tag{10}$$

for $x < x'$. For $x > x'$, we interchange x and x' in (10). By the inversion theorem we obtain

$$G_T(x,t|x',t') = \frac{2}{l}\sum_{n=1}^{\infty}\sin\frac{n\pi x}{l}\sin\frac{n\pi x'}{l}\exp\left[\frac{-Nn^2\pi^2(t-t')}{l^2}\right] \tag{11}$$

which has been given in Ref. 15. This series converges rapidly for large values of Nt. For small values of Nt, an alternative form can be found by the method of images, or by inverting (10) through the use of Laplace transform tables, as

$$G_T(x,t|x',t') = \frac{1}{2[\pi N(t-t')]^{1/2}}\sum_{n=-\infty}^{\infty}\left\{\exp\left[-\frac{(2nl + x' - x)^2}{4N(t-t')}\right] - \exp\left[-\frac{(2nl - x' - x)^2}{4N(t-t')}\right]\right\} \tag{12}$$

Thus, Eq. (3) including the boundary conditions (4) is transformed into the integral Eq. (8).

Once $\phi(x,t)$ and $T(x,t)$ are obtained, the total heat flux can be calculated from the equation

$$q(x,t) = -4N\partial T/\partial x - \tfrac{4}{3}\partial\phi/\partial x \tag{13}$$

In steady state, the radiation potential remains in the same form as (5), but now it is a function of x only,

$$\phi(x) = \phi_1(x) + 3 \int_0^l T^4(x')G_\phi(x|x')dx' \qquad (14)$$

where $\phi_1(x)$ and $G_\phi(x|x')$ are given by (6) and (7). The temperature field can be obtained from (8) by setting $t \to \infty$ and performing the integration with respect to t',

$$T(x) = T_0 - (T_0 - T_l)\frac{x}{l} + \frac{1}{N}\int_0^l [\phi(x') - T^4(x')]G'_T(x|x')dx' \qquad (15)$$

where

$$G'_T(x|x') = x(l - x')/l \text{ for } x < x' \\ = x'(l - x)/l \text{ for } x > x' \qquad (16)$$

Equations (14) and (15) were solved numerically in Ref. 14 through the use of the total potential, defined by $\chi = -(3NT + \phi)$.

For the steady, one-dimensional problem, however, it is convenient to use the energy equation in the form,

$$d^2T/dx^2 = -(1/3N)d^2\phi/dx^2 \qquad (17)$$

Integrating (17) twice gives

$$T = T_0 - (T_0 - T_l)\frac{x}{l} - \frac{1}{3N}[\phi(0) - \phi(l)]\frac{x}{l} + \frac{1}{3N}[\phi(0) - \phi(x)] \qquad (18)$$

For very small values of N, the iteration on T from (18) may converge very slowly. In this case we may apply integration by parts to the integral in (14) to remove the factor $1/N$ in the right-hand side of (18)

$$T^4 = T_0^4 \cosh\beta x + 3N(T_0 - T) - 3N(T_0 - T_l)\frac{x}{l} - \phi_1(x) + \phi(0) - [\phi(0) - \phi(l)]\frac{x}{l} + \int_0^x \cosh\beta(x - x') \times \frac{dT^4}{dx'}dx' - \frac{3\cosh\beta x + h\beta\sinh\beta x}{(3 + h^2)\sinh\beta l + 2h\beta\cosh\beta l} \times \int_0^l T^4[\beta\cosh\beta(l - x') + h\sinh\beta(l - x')]dx' \qquad (19)$$

This equation can be conveniently solved by iterating on T^4. This technique can also be applied to the transient case.

Once the radiation potential is known, the heat flux can be calculated by the equation,

$$q = -4N\frac{dT}{dx} - \frac{4}{3}\frac{d\phi}{dx} = \frac{4N}{l}(T_0 - T_l) + \frac{4}{3l}[\phi(0) - \phi(l)] \qquad (20)$$

Equation (1) together with (8) for transient state and (15) for steady state are especially useful in discussing the relative importance of absorption and emission, and the general character of profiles of the temperature and the radiation potential. If $\phi \to T^4$, ϕ tends to be a linear function of x, and T tends to be equal to T_c (unless $l \to \infty$ also). Whenever emission is greater than absorption, the quantity $(\phi - T^4)$ produces a source effect in (1) for the radiation potential and a sink effect in (8) for the temperature. The reverse is true when absorption is greater than emission. Because of radiation slip at surfaces, the possibility that $T^4 > \phi$ should occur near the hotter surface and that $T^4 < \phi$ near the colder surface. Thus, in general, curves of ϕ vs x should appear somewhat in the shape of reverse S, and $T(x,t)$ can be lower than $T_c(x,t)$ only in the hotter region of the medium. For $t \to \infty$, $T_c(x,\infty)$ becomes a straight line and hence $T(x,\infty)$ should appear in general in an S-shape.

Numerical Solution and Results

Equations (5) and (8) for the transient state and (14) and (18) for the steady case were solved numerically by the method of successive approximations on a CDC-6400 computer for $T_0 = 1$ and various values of N, l, h (or ϵ), and T_l. The spline-fit approximation[17] was used for integration and interpolation. The advantage of this method is that mesh size can be changed whenever it is necessary. For instance, the integration along the space coordinate in the transient case needs smaller mesh size near the source points ($x \approx x'$) for small values of t. The explicit part on the right-hand side of (8) was taken as the first approximation of $T(x,t)$, i.e.,

$$T^{(1)}(x,t) = T_c(x,t) + \int_0^t \int_0^l \phi_1(x')G_T(x,t|x',t')dx'dt' \qquad (21)$$

Ten divisions on the time coordinate and twelve on the space coordinate were found as the minimum mesh sizes to yield satisfactory results. Each iteration required about 30 sec. For early times, two iterations were sufficient, but 6 iterations were required for $N = 0.1$, $t = 3$ and 15 for $N = 0.03$, $t = 4$. For $N < 0.01$, the convergence is rather slow. However, when the results for $N = 0.03$ were used as the first approximation to calculate T for $N = 0.01$, four or five iterations were sufficient. The iteration was terminated after n steps when $|T^{(n)}(x,t) - T^{(n-1)}(x,t)| \leq 0.5\%$ of $T^{(n-1)}(x,t)$.

To save the core storage of the machine the solution of (8) for $T(x,t)$ was performed by dividing t into several steps. First, $T(x,t)$ was calculated up to a small value of t, say t_1. $T(x,t)$ was then found for $t_1 < t \leq t_2$ by using $T(x,t_1)$ as the initial temperature,

$$T(x,t) = T_c(x,t) + \int_0^t \int_0^l [\phi(x',t') - T^4(x',t')]G_T(x,t|x',t')dx'dt' + \int_0^l T(x,t_1)G_T(x,t|x',0)dx' \qquad (22)$$

Similarly, $T(x,t)$ was calculated between $t_n < t < t_{n+1}$. Calculated results by this procedure checked well with those obtained directly from (8).

The steady-state numerical solutions for $\phi(x)$ and $T(x)$ were much simpler. The explicit part on the right-hand side of (18) was taken as the first approximation. The iteration was stopped at $|T^{(n)}(x) - T^{(n-1)}(x)| \leq 0.1\%$ of $T^{(n-1)}(x)$. The time of each iteration was less than 0.14 sec. To calculate T for $N < 0.01$ (19) was used and the temperature for $N = 0.01$ was taken as the first approximation.

Some of the calculated results of the radiation potential are shown in Fig. 2, those of the temperature in Figs. 3, 4, and 5, and those of the heat flux in transient state in Fig. 6. Calculated heat flux for steady state was found in excellent agreement with those reported in Refs. 1 and 4. This is well known and is therefore not presented. Some results obtained from the integro-differential equation by Viskanta and Grosh[1] are reproduced in Fig. 5. It is seen that they are in good agreement with those obtained from (18).

Discussions and Concluding Remarks

The calculated curves of radiation potential and temperature, as shown in Figs. 2–5, do exhibit the general appearances as pointed out earlier. This character is more pronounced for surfaces with low emissivity and large t. As t becomes smaller and smaller, curves of radiation potential tend to become from an inverse S-shape to nearly simple curve concave upward, indicating that more radiant energy is absorbed than emitted everywhere in the medium. Conse-

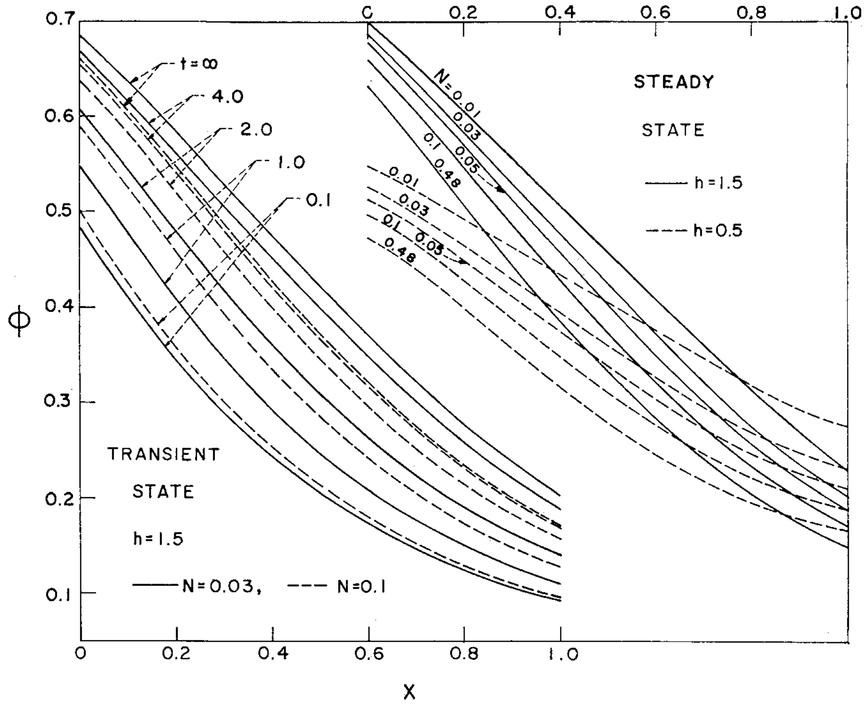

Fig. 2 Distribution of radiation potentials in transient and steady states, $T_0 = 1$, $T\ell = 0.1$, $l = 1.0$.

quently, for small t the temperature is everywhere higher than that of pure conduction, as shown in Figs. 3 and 4. Figure 2 shows that the gradient of radiation potential at $x = 0$ is always larger than that at $x = l$. Thus, the radiant energy is stored in the medium all the time. Figure 6 shows that radiation reduces the heat flow by conduction into the medium at earlier times but increases the conduction as the steady state is approached. The transient effect on the radiant heat flux at $x = 0$, i.e., $\Delta q_r = q_r(0,t) - q_r(0,\infty)$ is significant only for small values of t and decreases rapidly as t increases. Therefore, for $t > 1$ and $\epsilon > 0.5$, the total heat flux may be estimated by the simple equation

$$q(0,t) = q_c(0,t)_{\text{pure cond.}} + q_r(0,\infty) \tag{23}$$

Since the shapes of ϕ curves, as shown in Fig. 2, are insensitive to the variation of N and t for given values of the other parameters, there is great latitude in calculating the approximate heat flux at $x = 0$. For $t \to \infty$, the superposition of pure conduction and pure radiation is known to yield good results for black surfaces.[18] For $t < \infty$ we can calculate $q(0,t_1)$ with little error by using $[\partial T_c(x,t_1)/\partial x]_{x=0}$ and $[\partial\phi(x,t_2)/\partial x]_{x=0}$ for given values of the other parameters, provided that values of t_1 and t_2 are in the ranges: 0–0.1, 0.1–1, and 1–∞. Similarly, $q(0,t_1,N_1)$ can also be calculated with good approximation by

$$q(0,t_1,N_1) = -4N_1[(\partial/\partial x)T_c(x,t_1,N_1)]_{x=0} - \tfrac{4}{3}[(\partial/\partial x)\phi(x,t_1,N_2)]_{x=0} \tag{24}$$

where N_1 and N_2 are in the ranges: 0–0.01, 0.01–0.1, and 0.1–∞.

However, good approximate calculation of the temperature was found possible only for small values of t. As noted

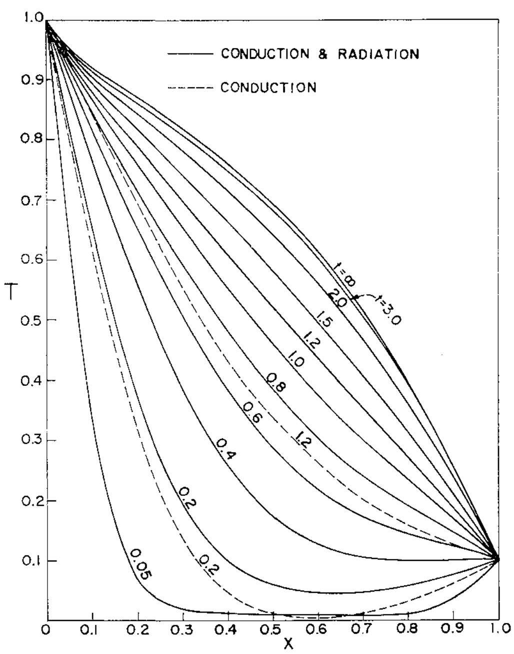

Fig. 3 Temperature distribution in transient state, $N = 0.1$, $h = 1.5$, $l = 1.0$.

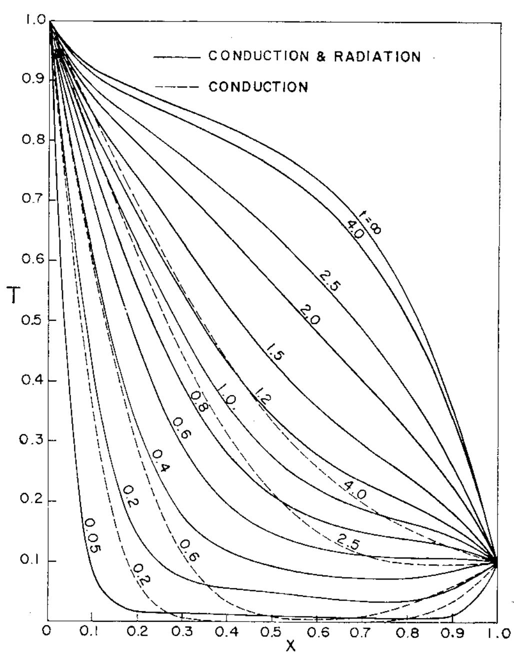

Fig. 4 Temperature distribution in transient state, $N = 0.03$, $h = 1.5$, $l = 1.0$.

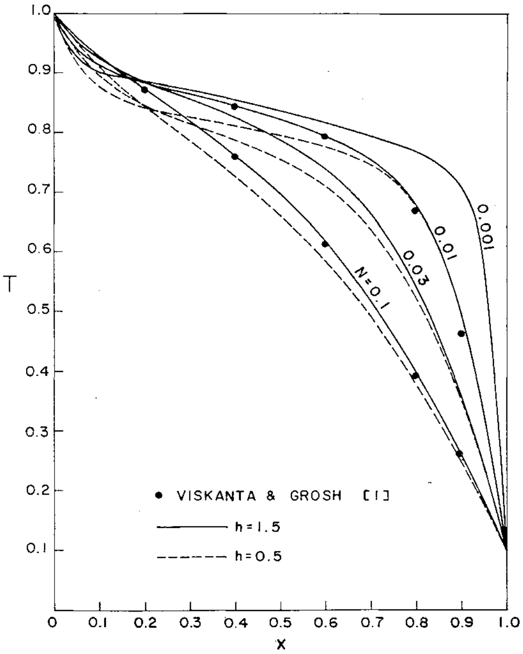

Fig. 5 Temperature distribution in steady state, $T_0 = 1.0$, $T_l = 0.1$, $l = 1.0$.

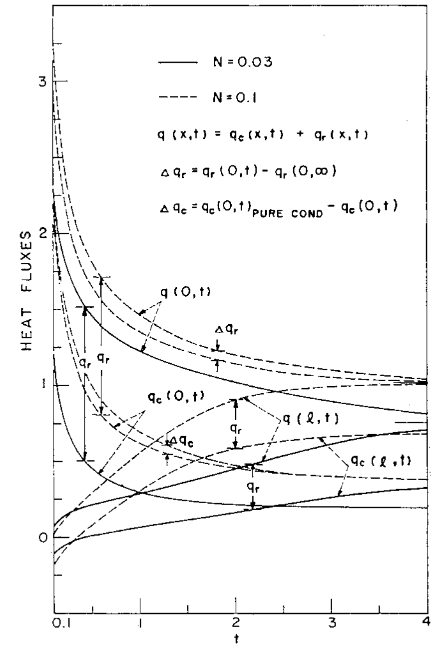

Fig. 6 Heat fluxes at surfaces in transient state, $T_0 = 1.0$, $T_l = 0.1$, $h = 1.5$, $l = 1.0$.

earlier, two iterations were sufficient for small t by using (21) as the first approximation. This indicates that, for small t, emission is indeed very small in comparison with absorption. For $t < 0.2$ and $N = 0.03$, or for $t < 0.4$ and $N = 0.1$, the error between $T(x,t)$ and $T^{(1)}(x,t)$ is less than 0.5% except near the surface where the error is still less than 1.0%. Thus, (21) is a good approximate solution valid for very small values of t. Obviously, if we substitute (6) and (12) into (21), an explicit expression of $T(x,t)$ can be obtained.

The temperature curves for $t = 1.0$ in Fig. 3 and for $t = 1.5$ in Fig. 4 appear in a somewhat wavy shape. This phenomenon occurs also in cylindrical medium.[19] Since this paper was submitted for publication, more calculations have been done for large values of l and various values of T_l. We have found that the larger the value of l, the more pronounced is the wavy shape and the larger is the value of t in which it occurs. For instance, it can be distinctly observed for $l = 2$, $T_l = 0$, and $N = 0.03$ at $t = 2.0$. When the cold surface is removed and $l \rightarrow \infty$, this phenomenon does not take place until $t \geq 1000$ for $N = 0.01$. This is qualitatively in agreement with what Lick[7] described as thermal waves.

Appendix

Under the assumptions stated earlier, the governing equations of radiation flux and average intensity, expressed in physical coordinates, were given in Refs. 10–13,

$$\mathbf{q}_r^* = -(4\pi/3\kappa)\nabla\bar{I}^* \tag{A1}$$

$$\nabla\cdot[(1/\kappa)\nabla\bar{I}^*] - 3\kappa\bar{I}^* = -3\kappa I_b^* \tag{A2}$$

To formulate the boundary conditions on $\bar{I}^*$, we follow Eddington by writing the average intensity in terms of two parts, one in the forward direction (denoted by a subscript +) and the other in backward direction (denoted by a subscript −) along the normal to the surface, so that

$$\bar{I}^* = (\bar{I}^*_+ + \bar{I}^*_-)/2 \tag{A3}$$

An energy balance on the surface gives

$$q_r^*(s) = \pi\bar{I}^*_+(s) - \pi\bar{I}^*_-(s) = \epsilon_s\pi I_{bs}^* - \epsilon_s\pi\bar{I}^*_- \tag{A4}$$

Applying (A1) to the surface and eliminating $q_r^*(s)$, $\bar{I}^*_+(s)$, and $\bar{I}^*_-(s)$ by (A3) and (A4), we obtain the boundary condition on $\bar{I}^*$ as

$$(\partial\bar{I}^*/\partial n)_s = h_s\kappa[\bar{I}^*(s) - I_{bs}^*] \tag{A5}$$

$$h_s = \frac{3\epsilon_s}{2(2-\epsilon_s)} \tag{A6}$$

where n is the inward-drawn normal to the surface. The energy equation for combined radiation and conduction can then be written as

$$\rho c\partial T^*/\partial t^* - \nabla\cdot(k\nabla T^*) = \tfrac{4}{3}\pi\nabla\cdot[(1/\kappa)\nabla\bar{I}^*] \tag{A7}$$

References

[1] Viskanta, R. and Grosh, R. J., "Heat Transfer by Simultaneous Conduction and Radiation in an Absorbing Medium," *Transactions of the ASME*, Vol. 84, Ser. C, 1962, pp. 63–72.

[2] Lick, W., "Energy Transfer by Radiation and Conduction," *Proceedings of Heat Transfer and Fluid Mechanics*, Stanford Univ. Press, 1963, pp. 14–23.

[3] Greif, R., "Energy Transfer by Radiation and Conduction with Variable Gas Properties," *International Journal of Heat and Mass Transfer*, Vol. 7, 1964, pp. 891–900.

[4] Wang, L. S. and Tien, C. L., "Study of the Interaction between Radiation and Conduction by a Differential Method," *Proceedings of the 3rd International Heat Transfer Conference*, Vol. 5, 1966, pp. 190–199.

[5] Einstein, T. H., "Radiant Heat Transfer to Absorbing Gases Enclosed Between Parallel Plates with Flow and Conduction," TR R-154, 1963, NASA.

[6] Howell, J. R., "Determination of Combined Conduction and Radiation of Heat Transfer Through Absorbing Media by the Exchange Factor Approximation," *Chemical Engineering Progress Symposium Series*, Vol. 61, No. 59, 1967, pp. 162–171.

[7] Lick, W., "Transient Energy Transfer by Conduction and Radiation," *International Journal of Heat and Mass Transfer*, Vol. 8, 1965, pp. 119–128.

[8] Nemchinov, I. V., "Some Non-stationary Problems of Radiation Heat Transfer," Transaction TT-4, School of Aerospace and Engineering Sciences, 1964, Purdue Univ.

[9] Viskanta, R. and Lall, P. S., "Transient Cooling of a Spherical Mass of High Temperature Gas by Thermal Radiation," *Journal of Applied Mechanics*, Vol. 32, Ser. E, 1965, pp. 740–746.

[10] Traugott, S. C., "A Differential Approximation for Radiative Transfer with Application to Normal Shock Structure," *Proceedings of Heat Transfer and Fluid Mechanics*, Stanford Univ. Press, 1963, pp. 1–13.

[11] Cohen, I. M., "Radiative Heat Flux Potential," *AIAA Journal*, Vol. 3, No. 5, May 1965, pp. 981–982.

[12] Cheng, P., "Two-dimensional Radiating Gas Flow by a Moment Method," *AIAA Journal*, Vol. 2, No. 9, Sept. 1964, pp. 1662–1664.

[13] Chang, Y. P., "A Potential Treatment of Energy Transfer by Conduction, Convection, and Radiation," *AIAA Journal*, Vol. 5, No. 5, May 1967, pp. 1024–1025.

[14] Chang, Y. P., "A Potential Treatment of Energy Transfer in a Conducting, Absorbing and Emitting Medium," ASME paper, 67/WA/HT-40, 1967.

[15] Carslaw, H. S. and Jaeger, J. C., *Conduction of Heat in Solid*, Oxford Press, Oxford, England, 1959, Chap. 14.

[16] Friedman, B., *Principles and Techniques of Applied Mathematics*, Wiley, New York, 1956, Chaps. 4 and 5.

[17] Katsanis, T., "Use of Arbitrary Quasi-orthogonels for Calculating Flow Distribution in the Meridianal Flow of a Turbomachine," TND-2546, 1964, NASA.

[18] Probstein, R. F., "Radiation Slip," *AIAA Journal*, Vol. 1, No. 5, May 1963, pp. 1202–1204.

[19] Chang, Y. P. and Smith, R. S., Jr., "Steady and Transient Heat Transfer by Radiation and Conduction in a Medium Bounded by Two Coaxial Cylindrical Surfaces," *International Journal of Heat and Mass Transfer*, Vol. 12, 1970, pp. 69–80.

APRIL 1970 AIAA JOURNAL VOL. 8, NO. 4

Measurement of $O_2^+ + e^-$ Dissociative Recombination in Expanding Oxygen Flows

MICHAEL G. DUNN* AND JOHN A. LORDI†

Cornell Aeronautical Laboratory Inc., Buffalo, New York

The dissociative-recombination rate coefficient for the reaction $O_2^+ + e^- \xrightarrow[k_r]{} O + O$ has been measured in the invisid nozzle flow of a short-duration reflected-shock tunnel and found to be given by $k_r = (8 \pm 2) \times 10^{21}\ T_e^{-1.5}$ cm³/mole sec for an electron temperature range of approximately 1800°K to 5000°K. These experiments were performed in oxygen at equilibrium reservoir conditions of 4950°K and 25 atm pressure. Thin-wire Langmuir probes were used to measure the electron temperature and electron density on the nozzle centerline. The electron densities were simultaneously measured using microwave interferometers.

1. Introduction

PREVIOUS papers[1,2] by the authors have presented rate-coefficient data for the dissociative recombination (k_r) of NO^+ and N_2^+. The purpose of this paper is to present rate-coefficient data, obtained in the same manner, for the reaction

$$O_2^+ + e^- \underset{k_i}{\overset{k_r}{\rightleftharpoons}} O + O$$

It is typical of this class of reactions that the ionization and recombination rate coefficients are related to the local plasma conditions through the temperature, the ionization rate being dependent upon the heavy-particle translational temperature while the recombination rate depends mainly on the electron temperature. Both of these temperatures must be known in experiments to evaluate the recombination-rate coefficient.

Experimental data for the two-body dissociative recombination of O_2^+ for electron temperatures greater than 450°K are scarce, with only two points reported.[3,4] However, many data points have been reported at 300°K or slightly greater temperatures. Discharge tube measurements, the only ones available, of the dissociative recombination of O_2^+ have been complicated as a result of the reported[5] presence of O_3^+ in nontrivial quantities. Sayers[3] and Sayers and Kerr[4] provide the only reports of high-temperature experiments, reporting values at approximately 2000°K and 2600°K. They studied the electron-density decay in the afterglow of a discharge. A radio-frequency mass spectrograph was used to identify the ions present in the plasma and to confirm the absence of negative ions.

Anisimov, Vinogradov, and Golant[6] measured the dissociative recombination of O_2^+ by studying the electron-density decay, using a microwave cavity resonator, in the afterglow of a discharge tube. A pulsed electrode discharge was used to create the plasma in a long chamber, along the axis of which they placed a strong magnetic field in an attempt to limit the effects of diffusion.

Biondi, Connor, and Weller[7] and Kasner and Biondi[8] studied the afterglow of an arc discharge and obtained data points at 300°K that were in reasonably good agreement with the other 300°K data.

Holt[9] investigated the dissociative recombination of O_2^+ in the afterglow decay of a plasma created by an arc discharge. He worked in the pressure range 0.3 to 10 mm Hg and detected electron densities in the range 10^8 to 10^{11} e^-/cm^3. The electron losses due to recombination, diffusion, and attachment for 300°K electrons were separated according to the respective loss laws. The experimental value of the reaction rate constant was in good agreement with others available for 300°K.

In the present experiments, both the electron temperature and the electron density were measured in the expanding oxygen plasma. Then the recombination rate coefficient for the dominant reaction was adjusted until the calculated number density agreed with the probe and microwave-interferometer data. The measured electron-temperature history was used in calculating the variation of the number density along the nozzle. The relative importance of various reactions included in the reaction model was independently assessed. In Sec. 2 the experimental apparatus and procedure are briefly dis-

Received July 3, 1969; revision received October 23, 1969. This research was supported by NASA, Goddard Space Flight Center, Greenbelt, Md., under Contract NAS 5-9978.

* Principal Engineer, Aerodynamic Research Department. Member AIAA.

† Research Aerodynamicist, Aerodynamic Research Department. Member AIAA.